Recent developments in semiclassical gravity

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Semiclassical gravity equations

- Consider quantum fields in a curved spacetime, with energy-momentum tensor T_{ab}. Semiclassical gravity is the approximation in which their back-reaction on the spacetime is governed by the equations

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- The metric is classical and the matter is quantum. The matter could also include gravitons.

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- Can also be derived in quantum gravity as the 1-loop perturbative correction to the expectation value of the metric in an expansion about a background classical solution. Energy-momentum tensor can include graviton contribution.

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- In this talk, we will specifically be interested in recent results that use semiclassical gravity. This will broadly include results where the metric is taken to be classical, and the matter quantum.

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$$S_{gen} = < \frac{A}{4\hbar G} > + S_{out}. \quad (1)$$

- Bekenstein proposed that S_{out} was the ordinary thermodynamic entropy of matter outside the horizon. He also proposed the Generalized second law, the statement that the generalized entropy never decreases in time.

- What if we have quantum fields outside the horizon? What should we take S_{out} to be? Sorkin (1983) pointed out it should be the entanglement entropy $-Tr(\rho \log \rho)$ where ρ is the reduced density matrix of the black hole exterior. However, this has a UV divergence.

- What if we have quantum fields outside the horizon? What should we take S_{out} to be? Sorkin (1983) pointed out it should be the entanglement entropy $-Tr(\rho \log \rho)$ where ρ is the reduced density matrix of the black hole exterior. However, this has a UV divergence.
- There is now a lot of evidence for the following: The leading divergent term can be absorbed in the renormalization of the bare Newton's constant G and the generalized entropy is finite and now contains the renormalized G. (Starting with Susskind/Uglum in 1993). Here, S_{ent} is the finite part of the entanglement entropy.

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- Recent developments supply such a renormalization scheme. We will review these recent developments.

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- They showed that generalized entropy of (the Kruskal extension of) the Schwarzschild black hole at its bifurcation surface is the entropy of this algebra modulo a constant.
- They also proved a version of a generalized second law for Einstein gravity using techniques from von Neumann algebras, specifically that the generalized entropy increases from early to late times for asymptotically AdS black holes, if the black hole is allowed to equilibrate in between the early and late times. This is not quite a *local* GSL which would be the statement $\frac{dS_{gen}}{dv} \geq 0$.

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- This computation generalizes to black holes in an arbitrary diffeomorphism invariant theory (Ali/VS, PRD 2023).

The algebra of observables

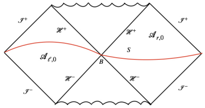

Figure: The algebra of the black hole exterior

Consider QFT on the maximally extended Kruskal black hole spacetime. The algebra of bounded smeared quantum field operators in the right exterior acting on the Hilbert space is denoted $\mathcal{A}_{r,0}$.

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- The algebra $\mathcal{A}_{r,0}$ of quantum fields in the right exterior is a von Neumann algebra. Its commutant is the algebra of operators in the left exterior $\mathcal{A}_{\ell,0}$. (Which is the largest causally disconnected region from the right exterior)

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- **'Entropy of an algebra'**: Type I and Type II von Neumann algebras have renormalizable density operators. Thus, one can define entropy for Type I and Type II algebras by considering the von Neumann entropy associated with the density operator.
- Type III von Neumann algebra has no renormalizable density operators.
- Renormalization is by defining a modified trace in situations where the Hilbert space trace is infinite.

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- We enlarge the algebra by adding an operator and enlarge the Hilbert space to get states on which this extra operator acts.
- Witten's idea: Consider a low energy effective field theory of quantum gravity - we have a matter QFT and gravitons in the Schwarzschild background.The algebra of fields in the black hole right exterior is Type III. We are interested in this theory in the $G \to 0$ limit.

- Add one more operator, the ADM Hamiltonian H_R and implement the constraint $H_R = \hat{h} + H_L$ where

$$\hat{h} = \int_S d\Sigma^\mu V^\nu T_{\mu\nu}$$

and V is the time translation Killing field of the Schwarzschild black hole. In Einstein gravity, $\hat{h} = H_R - H_L$. Actually, more precisely, we add the operator $H_R - M_0$, where M_0 is the mass of the original unperturbed black hole. This difference is finite in the $G \to 0$ limit.

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- Expand the Hilbert space to $\mathcal{H} \otimes L^2(\mathbb{R})$ where we have included square integrable functions of the conjugate variable, the timeshift $(t_L + t_R)$ on which the extra operator acts. Under the time translation symmetry, we have $t_L \to t_L + c$ and $t_R \to t_R - c$. Choose $c = t_R$, so we can set t_R to zero. The conjugate variable is then t_L.

- The operator $\beta\hat{h}$ in the QFT, is precisely the modular Hamiltonian associated with the Hartle-Hawking state of the QFT (Bisognano/Wichmann and Sewell). β is the inverse temperature of the black hole. The operator generates automorphisms of the algebra.
$e^{i\hat{h}s} a e^{-i\hat{h}s} \in \mathcal{A}_{r,0}, \qquad \forall\, a \in \mathcal{A}_{r,0},\ s \in \mathbb{R}.$

Definition of the crossed product algebra

- The crossed product left algebra is generated by operators $e^{it_L \hat{h}} a' e^{-it_L \hat{h}}, \; e^{is(H_L - M_0)}, \forall \; a' \in \mathcal{A}_{l,0} \; s \in \mathbb{R}$.

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- This is the crossed product by the modular automorphism group and the resultant algebra is Type II.
- By construction, the dressed operators $e^{it_L \hat{h}} a' e^{-it_L \hat{h}}$ commute with the constraint $\hat{h} + H_L - H_R$.

- The Type II algebra has a trace, a linear functional on $\mathcal{A}_r$ such that $tr(\hat{a}\hat{b}) = tr(\hat{b}\hat{a})$ and $tr(a^\dagger a) > 0$ for $a \neq 0$. This is not the Hilbert space trace, which may be infinite.

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- We can define a density matrix for any state of the extended Hilbert space $|\hat{\Phi}>$ by

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- Entropy of the algebra $\mathcal{A}_r$ for the state $\hat{\Phi}$ is

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- For the particular Type II algebra we started out with (a factor), tr is unique up to a scaling $tr \to e^c tr$ under which $S(\hat{\Phi})_{\mathcal{A}_r} \to S(\hat{\Phi})_{\mathcal{A}_r} + c$.

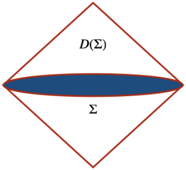
- Chandrasekaran/Penington/Witten (CPW): For **semiclassical** states $|\hat{\Phi}>$, the entropy of the algebra

$$S(\hat{\Phi})_{\mathcal{A}_r} = S_{gen} + C.$$

Here, S_{gen} is evaluated at the bifurcation surface B.

- We showed (Mohd Ali, V.S) that a similar result holds in any diffeomorphism invariant theory of gravity with the Area of the horizon replaced by the Wald entropy in S_{gen}.

Entropy of any subregion (Jensen/Sorce/Speranza)



- This construction was generalized by JSS to associate an algebra entropy to any subregion of spacetime which is a domain of dependence of some partial Cauchy slice. The algebra of observables in this subregion is the algebra of Einstein gravity + matter in the $G \to 0$ limit. If we just treat the graviton as one more quantum field, this is a type III algebra.

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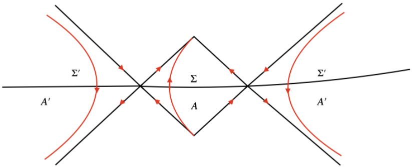
- JSS impose as a constraint, the Hamiltonian H generating the flow of a particular class of diffeomorphisms on a Cauchy slice, as the full gravity + matter theory is diffeomorphism invariant. This class of diffeomorphisms preserve the subregion and its causal complement. This is a generalization of the work of CPW who imposed the generator of an isometry as a constraint.



These diffeomorphisms must behave like the Killing isometries of Rindler space in the local Lorentz frame at the entangling surface.

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- We need the algebra of operators that commute with the constraint. This is given by the algebra $\{e^{iHp} a e^{-iHp}, e^{iqt}, t \in R\}''$. This has the structure of a crossed product algebra.
- A crucial assumption of JSS is that H (this is a local integral on some Cauchy slice) is a modular Hamiltonian associated to some state. Since the diffeomorphism is not an isometry, the value of H will depend on the choice of Cauchy slice.

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- Since the algebra is now type II, one has a trace on the algebra, renormalized density matrices, and an entropy for the algebra. The entropy is the generalized entropy modulo a constant, and that depends on the area of the entangling surface and the entanglement entropy of the partial Cauchy slice.

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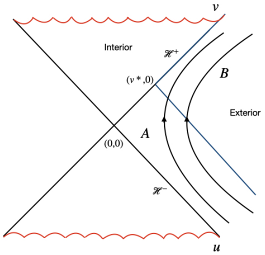
- However, there is one instance when the assumption of JSS is exactly true. That is when the subregions are specific wedges in black hole spacetimes.



The modular Hamiltonian (of the vacuum) for each such wedge can be computed.

A local GSL in crossed product constructions (Mohd Ali, V.S, 2024)

- This modular hamiltonian is the generator of the flow of a vector field which obeys the JSS conditions in the wedge. Hence we can use the JSS construction.

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- We use the relative entropy of two states in the type II algebra which is finite. We can show that for semiclassical states, this is the same as the relative entropy in the type III algebra. The observer degrees of freedom cancel out in the relative entropy in each wedge.

- The relative entropy is monotonic, consider the wedge at $v**$ which is contained in the one at $v*$,

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- Using this, we can show $S_{gen}(v**) \geq S_{gen}(v*)$ for all $v** \geq v* \geq 0$. This is a local GSL.

Open questions

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- Our proof of the GSL is valid for stationary black hole Killing horizons. What about more general dynamical horizons?
- Is there some monotonic ‘entropy’ and a GSL away from the semiclassical approximation (Kirklin, 2024)? Is the monotonic quantity the algebra entropy and can it be computed away from the semiclassical approximation?

An algebraic description of the information loss problem (van der Heijden, Verlinde, 2024)

- From the work of Page, we know that for an evaporating black hole, if we throw a diary into the black hole before the Page time, the information of the diary does not come out before the Page time.

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- Recently, the information recovery protocol has been described in the language of von Neumann algebras. How is there a transition from the Type I description to the emergent spacetime Type III description?

Thank you!